THEORY OF THE VARACTOR FREQUENCY HALVER

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ABSTRACT

A basic theory for broadband balanced frequency-halving circuits is presented. The analysis is applicable to both Schottky-barrier and reverse-biased abrupt-junction varactors and is based on the solution of exact nonlinear differential equations. An approximate algebraic method yields the global steady-state amplitude and phase solutions, while numerical integration gives transient solutions under specific conditions. These circuits are useful in bandwidth-compression, frequency-translation, PSK carrier-recovery and for stabilizing RF sources to LF references.

Introduction

Although recent papers have described practical frequency halvers designed by various empirical/experimental[1-3] and computer-aided[4,5] techniques, no satisfactory theory has been available to account for their interesting large-signal and wideband properties. This paper gives the basic theory for a class of balanced frequency-halving networks which depend for their operation on the nonlinear capacitance-voltage relationship of abrupt-junction or Schottky-barrier varactors. Frequency- and time-domain solutions are presented.

For the steady-state response, the differential equation model is solved by an approximate analytical method because this gives a deeper insight into the nonlinear interactions than does a strictly numerical analysis. Such insight is considered important since the operation of these parametric subharmonic frequency-halvers is much less well understood than, for example, the operation of mixers. In the analysis, advantage is taken of the fact that the inverse half-power capacitance-voltage law typical of both types of varactor leads to considerable mathematical simplification and to the possibility of closed-form expressions for the steady-state solution.

Numerical integration is used to obtain the transient response because of the complexity of the corresponding analytical solution.

Halver Topology

Previously-described balanced halvers have employed the topologies of Fig. 1(a,b). In each case an inductive loop contains two varactors. An input at 2ω excites both varactors in phase. Under specific conditions, balanced loop oscillations occur at a frequency ω, at which the varactor voltages are 180^o out of phase. Since the amplitude increases towards a large final value and the circuit is strongly nonlinear, conventional analyses cannot be used.

Fig. 2 shows a simple model for the practical networks. The U-shaped microstrip loop of Fig. 1(a) and the transverse waveguide section of Fig. 1(b) are represented by the centre-tapped inductor L_1. Each varactor has a cutoff frequency

$$f_c(V_b) = \frac{1}{2\pi r_s C_j(V_b)} \tag{1}$$

where $C_j(V_b)$ is the depletion-region capacitance and r_s is the diode series resistance. In Fig. 2, L_2 and R_L represent the coupling of the subharmonic voltage which appears across AA' (Fig.1(a,b)) to the external load.

For a threshold input level P_{in}, the loop resonates at a particular subharmonic frequency

$$\omega_o(V_b) = [\tfrac{1}{2} L_1 C_j(V_b)]^{-\frac{1}{2}} \tag{2}$$

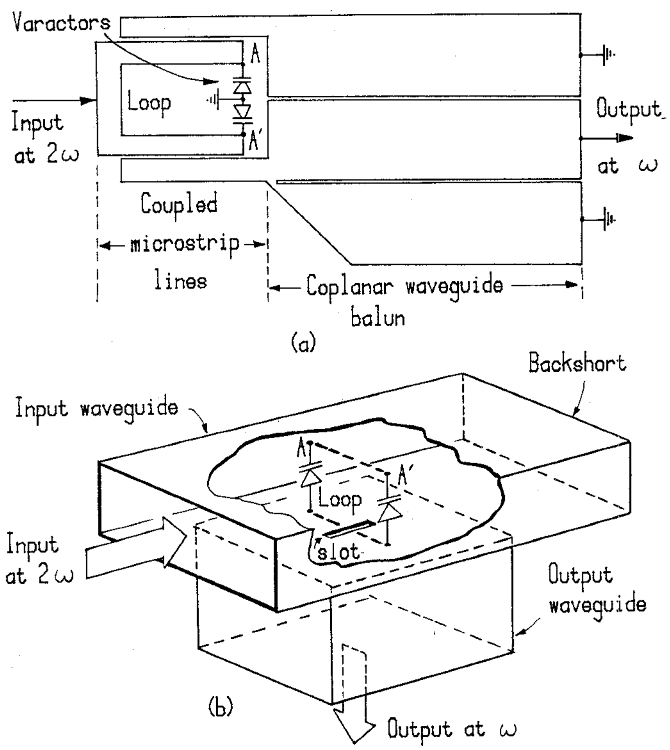

Fig. 1(a) Basic microstrip/CPW frequency-halving network[1].
(b) Basic waveguide halving structure[4].

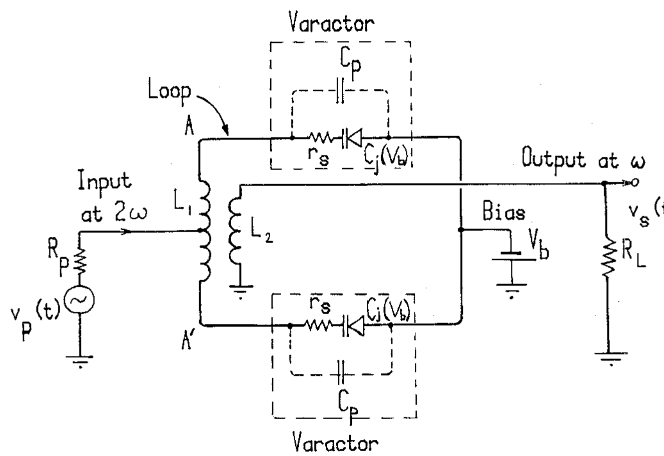

Fig. 2 Model for the basic wideband frequency halver.

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For increasing P_{in}, 1/2-subharmonic output occurs over increasing bandwidths, the greater increase being for $\omega<\omega_o$. For sufficient P_{in}, octave-plus bandwidth is possible.

Mathematical Model

The varactors in Fig. 2 are modelled as

$$\frac{v(t)}{V_o} = (1 + \frac{q(t)}{Q_o})^2 - 1 \qquad (3)$$

where

$V_o = \phi_o + V_b$

ϕ_o = built-in potential

V_b = reverse bias voltage

$Q_o = 2V_oC_j(V_b)$.

This is valid for both Schottky and reverse-biased* abrupt-junction varactors provided

$$\frac{v(t)}{V_o} > -1 \qquad (4)$$

and permits an exact differential equation model to be found when the inductors are tightly coupled:

$$(\xi + \xi_p)\dot u + u + \tfrac{1}{4}(u^2 + z^2) = x(t) \qquad (5a)$$

$$\ddot z + \xi\dot z + z + \tfrac{1}{4}uz = \dot y/\xi_L \qquad (5b)$$

$$\ddot z = \tfrac{1}{2}y + \dot y/\xi_L \quad . \qquad (5c)$$

All derivatives are with respect to $\tau = \omega_o t$ and the normalizations are:

$u = (q_1 + q_2)/Q_o$, sum of varactor charges

$z = (q_1 - q_2)/Q_o$, difference between them

$x = v_p(t)/V_o$, pumping voltage

$\xi_p = \omega_o R_p \cdot C_j(V_b)$, pump damping

$\xi_L = \omega_o R_L C_j(V_b)$, load damping

$\xi = \frac{\omega_o}{2\pi f_c(V_b)}$, ratio of resonance to cutoff frequencies.

Equation (5a) describes the input circuit, (5b) the resonant loop and (5c) the output circuit. Equations (5a) and (5b) are coupled only through their nonlinear terms; the corresponding circuits are coupled only via the varactor nonlinearities.

Analytical Solution

For halving, the input is

$$x(\tau) = X\cos 2\nu\tau, \qquad (6)$$

where $\nu = \omega/\omega_o$, the detuning. By symmetry, $z(\tau)$ and $y(\tau)$ contain only odd harmonics, whereas $u(\tau)$ contains only even harmonics. Analysis is simplified by assuming R_L large (ξ_L^{-1} small). Then eqns. (5) reduce to a single differential equation in z:

$$(\ddot z)^2 - 2(\xi+\xi_p)z(\ddot z+\xi\dot z) + 2(2\xi+\xi_p)\dot z\ddot z + \xi(3\xi+2\xi_p)(\dot z)^2 + \frac{1}{4}z^4 = z^2(1+x). \qquad (7)$$

Following Hayashi[6], an approximate ½-subharmonic solution is found by replacing $z(\tau)$ by its fundamental component $\tilde z(\tau) = Z\cos(\nu\tau+\psi)$, (8)

harmonics being neglected. By substituting (8) into (7) and equating phase and quadrature components separately to zero, steady-state solutions for the differential charge amplitude Z and phase ψ are obtained. The corresponding output voltage is found by using (5c), i.e.

$$\tilde y(\tau) = Y\cos(\nu\tau + \theta) = 2\ddot{\tilde z} \qquad (9)$$

This implies that

$$Y = -2\nu^2 Z \qquad (10)$$

and $\theta = \psi$. (11)

The resulting solution for the subharmonic amplitude Y is

$$[\tfrac{3}{2}(\nu^4-1) + \tfrac{1}{2}\xi(9\xi+8\xi_p)\nu^2 + \frac{5}{64}\cdot\frac{Y^2}{\nu^4}]^2 + [2\xi\nu]^2 = X^2 \qquad (12)$$

with phase angle

$$\theta = \frac{1}{2}\arcsin(\frac{2\xi\nu^3}{X}) \quad . \qquad (13)$$

The validity condition (4), together with (10), imposes the constraint

$$Y \leq 4\nu^2 \qquad (14)$$

since $\max |Z| = 2.0$. (15)

Steady-State Response

The limit of frequency-halving on the (ν,X)-plane is found by setting Y=0 in (12). An example for $\xi = 0.1$, $\xi_p = 0$ is shown in Fig. 3.

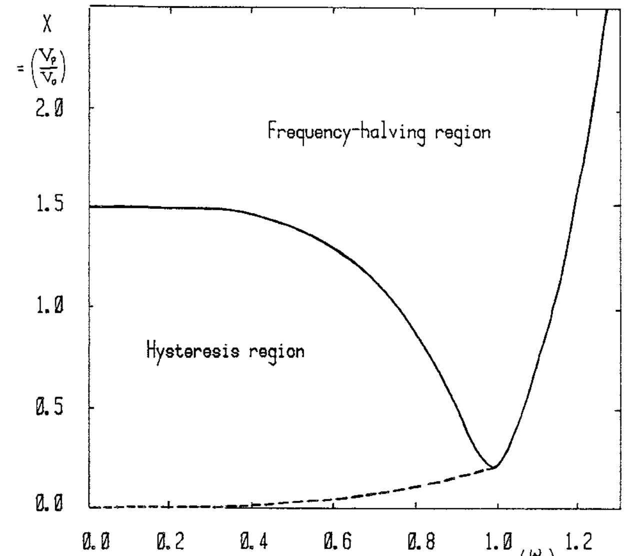

Fig. 3 Calculated frequency-halving and hysteresis regions for $\xi = \frac{\omega_o}{2\pi f_c} = 0.1$, $\xi_p = 0$, $\xi_L^{-1} = 0$. Small-signal subharmonic resonance occurs at $\nu = 1.0$.

In agreement with experiment[1], the halving bandwidth increases with X (or P_{in}), the greater increase occuring for $\nu < 1$ (or $\omega < \omega_o$). Octave-plus bandwidth is obtained for X > 1.3**. Also shown is the limit of the hysteresis region. Experimentally, this region is smaller. The difference is due to the varactor parasitic C_p, neglected here.

Fig.4 shows (X,Y) - profiles for $0.9<\nu<1.1$ and $\xi = 0.1$.

* To avoid charge-storage phenomena.

** These results confirm the observed reduction in threshold level and resonance frequency as the magnitude of the reverse bias V_b is reduced [1, Fig.4].

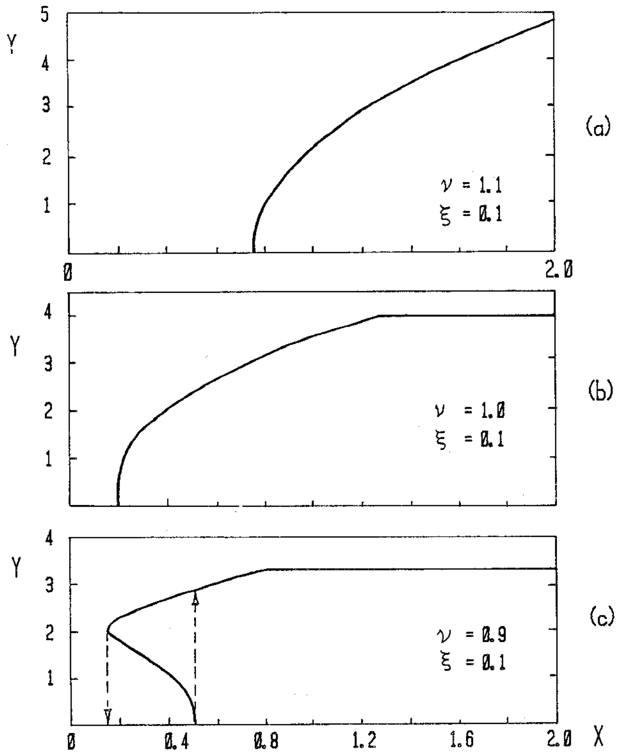

Fig. 4 Subharmonic output $Y(=V_s/V_o)$ versus input $X(=V_p/V_o)$ for 3 values of frequency $\nu(=\omega/\omega_o)$.

The flat regions in Fig. 4(b,c) correspond to constraint (14). Amplitude hysteresis is evident in Fig. 4(c).

A frequency-halving response surface in (ν,X,Y)-space is shown in Fig. 5 for the case $\xi = 0.1$.

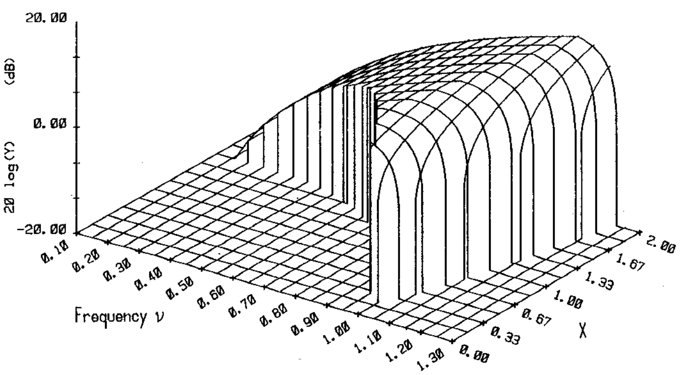

Fig. 5 Frequency-halving response surface in (ν,X,Y)-space for ξ=0.1. Hysteresis curves omitted for clarity.

Numerical Solution

Whereas the analytical solution approximates the global response of the halver, numerical integration of the differential equations (5) provides accurate transient and steady-state solutions under specific conditions. For a given set of values of ν, X, ξ, ξ_P and ξ_L the nature of the transient depends on three initial conditions: $u(0), z(0)$ and $\dot{z}(0)$. In the hysteresis region they determine the presence or absence of a final steady-state solution. The typical result of Fig. 6 shows (normalized) (a) the pump voltage $x(\tau)$, (b) the sum charge $u(\tau)$, (c) the difference charge $z(\tau)$ and (d) the output voltage $y(\tau)$. The envelope of $y(\tau)$ resembles experimental results [e.g. Fig. 9c of [1]]. Fig. 6(b) shows that the sum charge varies primarily at the pump frequency, while Fig. 6(c) demonstrates the validity of approximation (8).

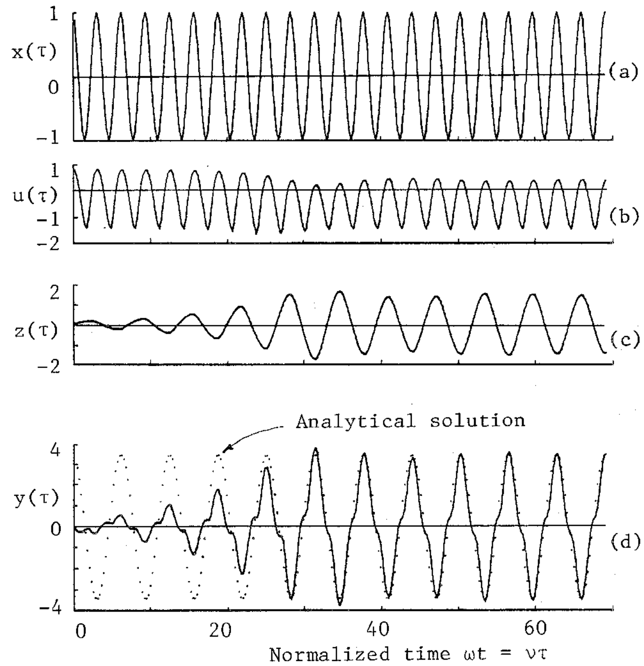

Fig. 6 Transient response for X=1.0,ν=1.0,ξ=0.1. Normalized quantities: (a) input at 2ω,(b) sum charge, (c) difference charge,(d) output voltage. Initial conditions: u(0)= 0.5,z(0)=0.0 ,z(0)= 0.2.

Conclusions

A basic theory for the broadband frequency halver has been developed. It accounts for the significant characteristics of various experimental realizations and provides a starting-point for detailed simulations including parasitics and distributed topologies.

Acknowledgements

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